

## Geometry 11.4 Notes: Circumference & Arc Length (pp 683–5)

### THEOREM

#### THEOREM 11.6 *Circumference of a Circle*

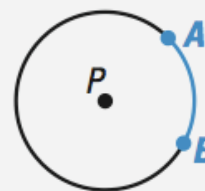
The circumference  $C$  of a circle is  $C = \pi d$  or  $C = 2\pi r$ , where  $d$  is the diameter of the circle and  $r$  is the radius of the circle.

An **arc length** is a portion of the circumference of a circle. You can use the measure of the arc (in degrees) to find its length (in linear units).

### COROLLARY

#### ARC LENGTH COROLLARY

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to  $360^\circ$ .



$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360^\circ}, \text{ or Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$$

#### EXAMPLE 1 *Using Circumference*

Find (a) the circumference of a circle with radius 6 centimeters and (b) the radius of a circle with circumference 31 meters. Round decimal answers to two decimal places.

#### SOLUTION

a.  $C = 2\pi r$   
 $= 2 \cdot \pi \cdot 6$   
 $= 12\pi$       Use a calculator.  
 $\approx 37.70$

► So, the circumference is about 37.70 centimeters.

b.  $C = 2\pi r$   
 $31 = 2\pi r$   
 $\frac{31}{2\pi} = r$       Use a calculator.

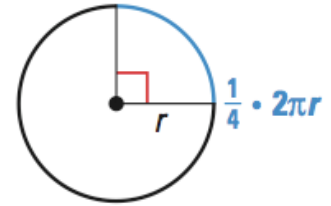
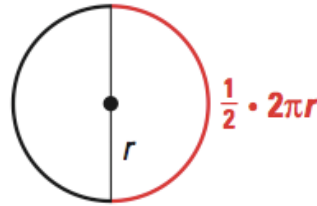
$$4.93 \approx r$$

► So, the radius is about 4.93 meters.

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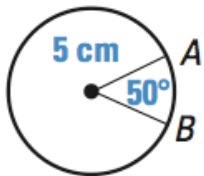
The length of a **semicircle** is one half the circumference, and the length of a **90° arc** is one quarter of the circumference.



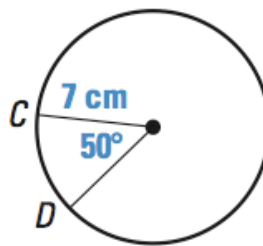
### EXAMPLE 2 Finding Arc Lengths

Find the length of each arc.

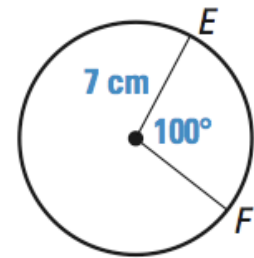
a.



b.



c.

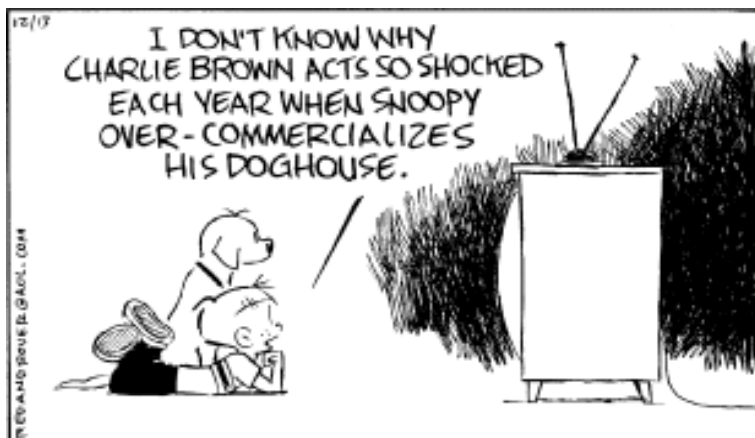


### SOLUTION

a. Arc length of  $\widehat{AB} = \frac{50^\circ}{360^\circ} \cdot 2\pi(5) \approx 4.36$  centimeters

b. Arc length of  $\widehat{CD} = \frac{50^\circ}{360^\circ} \cdot 2\pi(7) \approx 6.11$  centimeters

c. Arc length of  $\widehat{EF} = \frac{100^\circ}{360^\circ} \cdot 2\pi(7) \approx 12.22$  centimeters

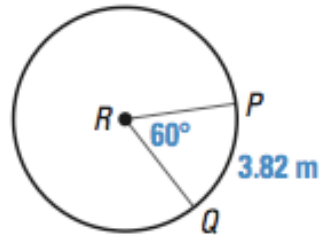


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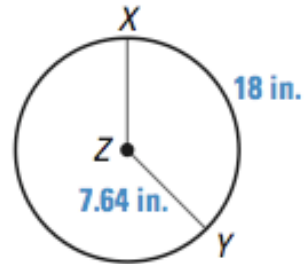
## EXAMPLE 3 Using Arc Lengths

Find the indicated measure.

a. Circumference



b.  $m\widehat{XY}$



### SOLUTION

$$\text{a. } \frac{\text{Arc length of } \widehat{PQ}}{2\pi r} = \frac{m\widehat{PQ}}{360^\circ}$$

$$\frac{3.82}{2\pi r} = \frac{60^\circ}{360^\circ}$$

$$\frac{3.82}{2\pi r} = \frac{1}{6}$$

$$3.82(6) = 2\pi r$$

$$22.92 = 2\pi r$$

► So,  $C = 2\pi r \approx 22.92$  meters.

$$\text{b. } \frac{\text{Arc length of } \widehat{XY}}{2\pi r} = \frac{m\widehat{XY}}{360^\circ}$$

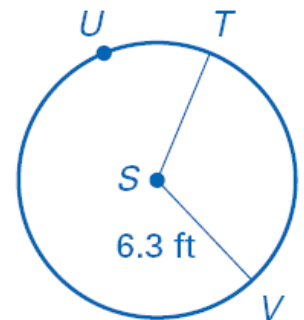
$$\frac{18}{2\pi(7.64)} = \frac{m\widehat{XY}}{360^\circ}$$

$$360^\circ \cdot \frac{18}{2\pi(7.64)} = m\widehat{XY}$$

$$135^\circ \approx m\widehat{XY}$$

► So,  $m\widehat{XY} \approx 135^\circ$ .

**6. Guided Practice:** Find  $m\widehat{TUV}$  if the arc length of  $\widehat{TUV}$  is 27 feet.



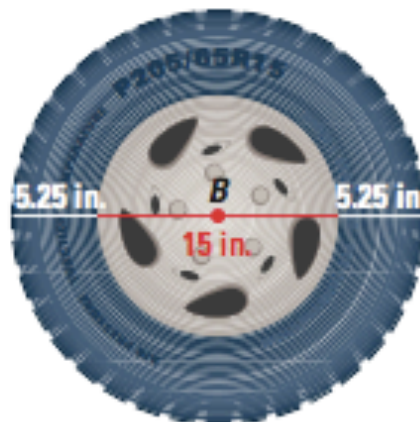
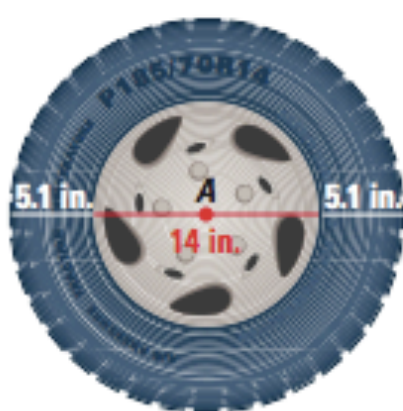
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### GOAL 2 CIRCUMFERENCE CIRCUMFERENCES

#### EXAMPLE 4 *Comparing Circumferences*



**TIRE REVOLUTIONS** Tires from two different automobiles are shown below. How many revolutions does each tire make while traveling 100 feet? Round decimal answers to one decimal place.



#### **SOLUTION**

Tire A has a diameter of  $14 + 2(5.1)$ , or 24.2 inches. Its circumference is  $\pi(24.2)$ , or about 76.03 inches.

Tire B has a diameter of  $15 + 2(5.25)$ , or 25.5 inches. Its circumference is  $\pi(25.5)$ , or about 80.11 inches.

Divide the distance traveled by the tire circumference to find the number of revolutions made. First convert 100 feet to 1200 inches.

$$\begin{aligned}\text{Tire A: } \frac{100 \text{ ft}}{76.03 \text{ in.}} &= \frac{1200 \text{ in.}}{76.03 \text{ in.}} \\ &\approx 15.8 \text{ revolutions}\end{aligned}$$

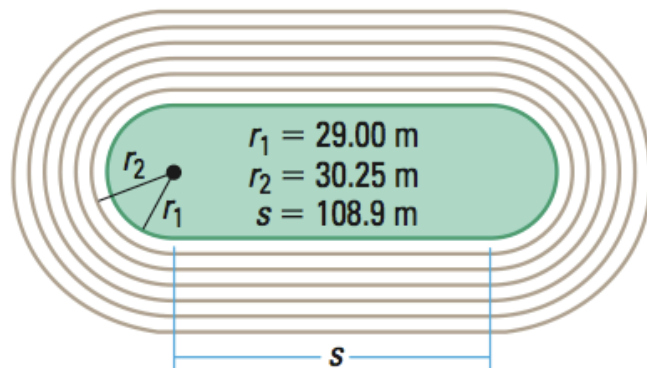
$$\begin{aligned}\text{Tire B: } \frac{100 \text{ ft}}{80.11 \text{ in.}} &= \frac{1200 \text{ in.}}{80.11 \text{ in.}} \\ &\approx 15.0 \text{ revolutions}\end{aligned}$$

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### EXAMPLE 5 *Finding Arc Length*

**TRACK** The track shown has six lanes. Each lane is 1.25 meters wide. There is a  $180^\circ$  arc at each end of the track. The radii for the arcs in the first two lanes are given.

- Find the distance around Lane 1.
- Find the distance around Lane 2.



### SOLUTION

The track is made up of two semicircles and two straight sections with length  $s$ . To find the total distance around each lane, find the sum of the lengths of each part. Round decimal answers to one decimal place.

a. Distance =  $2s + 2\pi r_1$   
 $= 2(108.9) + 2\pi(29.00)$   
 $\approx 400.0$  meters

b. Distance =  $2s + 2\pi r_2$   
 $= 2(108.9) + 2\pi(30.25)$   
 $\approx 407.9$  meters

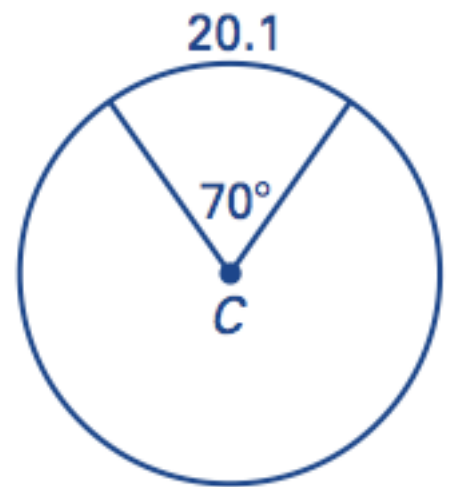
### Guided Practice.

10. A tire has an inside diameter of 22 in. and is 5 in. thick. How many revolutions will it make while driving 1 mi? Hint: 1 mile = 5280 feet

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19. \_\_\_\_\_ Find the radius of  $\odot C$ . Round to the nearest tenth.

- A. 16.5
- B. 32.9
- C. 51.7
- D. 65.8
- E. 103.4



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